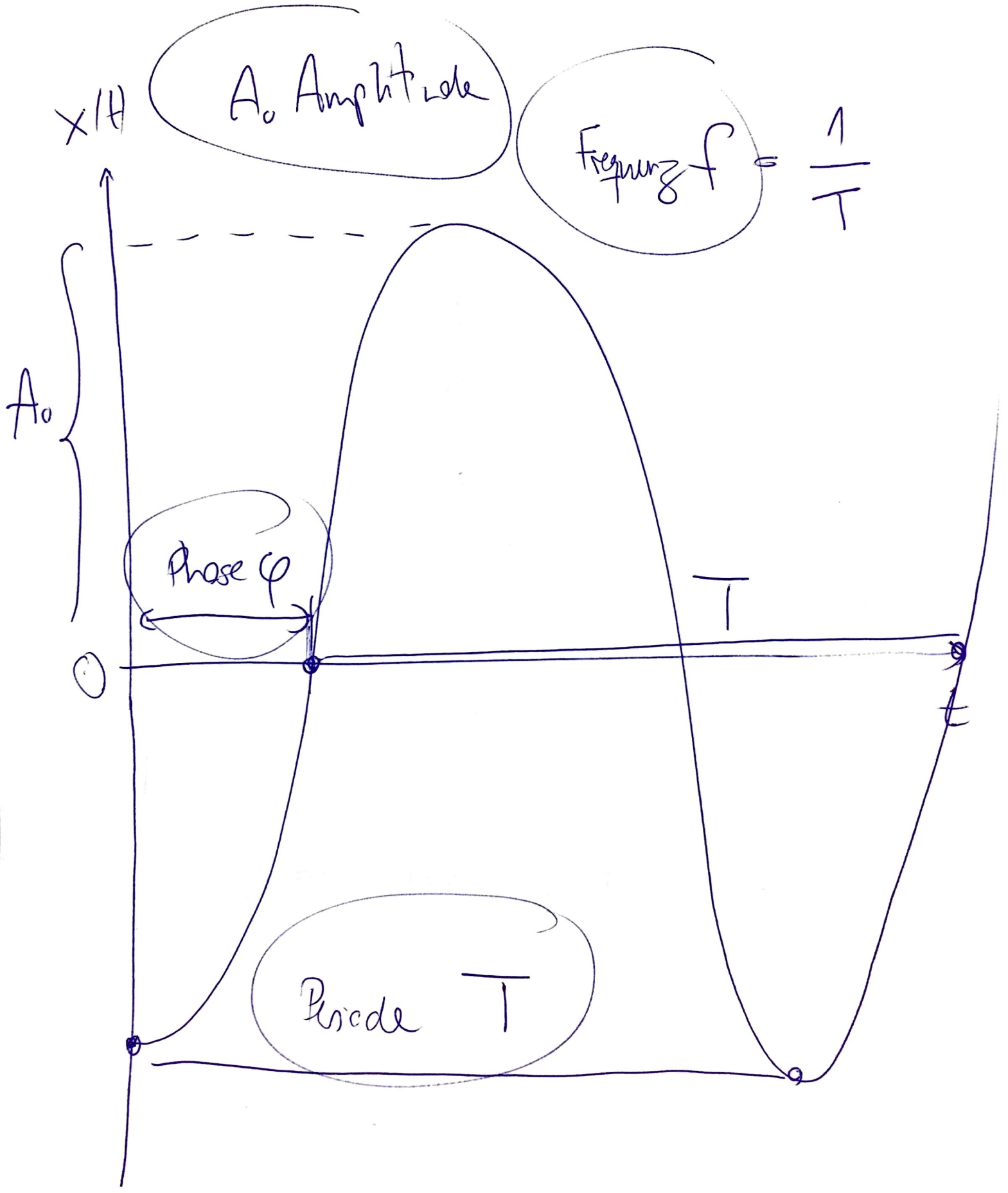
A₀ Amplitude
Frequenz f = 1/T
x(t)
A₀
Phase φ
T
0
t
Periode T

$F = -Kx$ Hooke (gegeben)

$F = m \cdot a$ Newton (gilt immer)

$$-Kx = m \cdot a$$

$$-Kx = m \cdot \frac{d^2}{dt^2} x$$

$$-Kx = m \ddot{x}$$

DGL

$$-K\,x(t) = m \cdot \frac{d^2}{dt^2} x(t)$$

$$\frac{d^2}{dt^2} x(t) + \frac{K}{m} \cdot x(t) = 0$$

$$\frac{d^2}{dt^2} x(t) + \frac{K}{m} x(t) = 0$$

$$x(t) = \sin \omega t$$

$$\dot{x}(t) = \omega \cos \omega t$$

$$\ddot{x}(t) = -\omega^2 \sin \omega t$$

$$-\omega^2 \sin \omega t + \frac{K}{m} \sin \omega t = 0$$

$$\left(\frac{K}{m} - \omega^2\right) \sin \omega t = 0$$

$$\Rightarrow \quad \frac{K}{m} - \omega^2 = 0$$

$$\Rightarrow \quad \frac{K}{m} = \omega_1^2, \qquad \omega_1 = \sqrt{\frac{K}{m}}$$

$$\frac{K}{2m} = \omega_2^2, \qquad \omega_2 = \sqrt{\frac{K}{2m}} = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{k}{m}}$$

$$= \frac{1}{\sqrt{2}} \cdot \omega_1$$

$$x(t) = A_0 \cdot e^{-\alpha \cdot t}$$

$$\frac{d}{dt} x(t) = -\alpha A_0 e^{-\alpha t}$$

$$\frac{d^2}{dt^2} x(t) = \alpha^2 A_0 e^{-\alpha t}$$

$$= \alpha^2 x(t)$$

$$\frac{d}{dt^2} A_0 e^{-\alpha t} + \frac{K}{m} A_0 e^{-\alpha t}$$

$$\alpha^2 \cdot x(t) + \frac{K}{m} x(t) = 0$$

$$\left(\alpha^2 + \frac{K}{m}\right) x \cdot (t) = 0$$

$$x(t) = A \cdot \cos(\omega t + \varphi)$$

$$\dot{x}(t) = -\omega A \sin(\omega t + \varphi)$$

Randbed. 1

$$\boxed{x(0) = A_0}$$

$$x(0) = A \cdot \cos(\varphi) = A_0$$

Randbed 2

$$v(0) = \boxed{\dot{x}(0) = 0}$$

$$\dot{x}(0) = -\omega A \cdot \sin(\varphi) = 0$$

$$\Rightarrow \varphi = 0$$

$$A \cdot \overbrace{\cos(0)}^{=1} = A_0$$

$$A = A_0$$

$$x(t) = A_0 \cdot \cos \omega t \text{ mit } \begin{cases} A = A_0 \\ \varphi = 0 \end{cases}$$

$$x(t) = A\cos(\omega t + \varphi)$$

1. Randbed. $x(0) = 0$

2. Randbed. $\dot{x}(0) = v_0$

1. 2.

1. $x(0) = A \cdot \cos\varphi \overset{!}{=} 0$

$$\Rightarrow \varphi = \pi/2$$

2. $\dot{x}(0) = v_0 = -A\omega \sin\frac{\pi}{2}$

$$= -A\omega$$

$$\Rightarrow A = -\frac{v_0}{\omega}$$

$$x(t) = -\frac{v_0}{\omega}\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$= -\frac{v_0}{\omega}\sin\omega t$$